

Photoproduction of η_c in NRQCD

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Abstract

We present a calculation for the photoproduction of η_c under the framework of NRQCD factorization formalism. We find a quite unique feature that the color-singlet contribution to this process vanishes at not only the leading order but also the next to leading order perturbative QCD calculations and that the dominant contribution comes from the color-octet $^1S_0^{(8)}$ subprocess. The nonperturbative color-octet matrix element of $^1S_0^{(8)}$ is related to that of $^3S_1^{(8)}$ of J/ψ by the heavy quark spin symmetry, and the latter can be from the direct production of J/ψ at large transverse momentum at the Fermilab Tevatron. We then conclude that the measurement of this process may clarify the existing conflict between the color-octet prediction and the experimental result on the J/ψ photoproduction.

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Studies of heavy quarkonium production in high energy collisions may provide important information on both perturbative and nonperturbative QCD. It is well known from compelling theoretical [1,2] and phenomenological [3] reasons that the Color Singlet model (CSM) used to describe the production and decay of quarkonium is not complete. In the CSM, it is assumed that the $c\bar{c}$ must be produced in a color-singlet state with the same angular-momentum quantum number as the charmonium meson which is eventually observed. However, it is well known that perturbative QCD calculations of production and decay of P-wave quarkonia within the CSM are plagued by infrared divergences [4]. More recently, measurements made at the Fermilab Tevatron show that the CSM also fails to predict the production cross sections of S-wave quarkonia (e.g. J/ψ and ψ') [3,5]. The naive CSM has been supplanted by the NRQCD factorization formalism [2], which allows the infrared safe calculation of inclusive charmonium production and decay rates. In this approach, the production process is factorized into short and long distance parts, while the latter is associated with the nonperturbative matrix elements of four-fermion operators. It also predicts a new mechanism called the color-octet mechanism in which a $c\bar{c}$ pair is produced at short distances in a color-octet state, and hadronizes into a final state charmonium nonperturbatively. This color-octet mechanism might provide an explanation for the Tevatron data on the surplus of J/ψ and ψ' production [6,7].

Even though the color-octet mechanism has achieved some successes in describing the production and decay of heavy quarkonia, more tests of this mechanism are still needed. Recently, the photoproduction data from HERA [8] puts a question about the universality of the color-octet matrix elements [9,10], in which the fitted values of the matrix elements $\langle\mathcal{O}_8^{J/\psi}(^1S_0)\rangle$ and $\langle\mathcal{O}_8^{J/\psi}(^3P_J)\rangle$ are one order of magnitude smaller than those determined from the Tevatron data [7]. (More recently, possible solutions for this problem have been suggested in [11,12]. In this situation it is certainly helpful to find other processes to test the color-octet mechanism in the heavy quarkonium production.

In this paper, we will calculate photoproduction of η_c , $\gamma p \rightarrow \eta_c + X$, under the framework of NRQCD. The photoproduction processes proceed predominantly through photon-gluon

fusion at high energies. In NRQCD the Fock state expansion for η_c is

$$\begin{aligned}
|\eta_c\rangle = & O(1)|c\bar{c}({}^1S_0, \underline{1})\rangle + O(v)|c\bar{c}({}^1P_1, \underline{8})g\rangle + O(v^2)|c\bar{c}({}^3S_1, \underline{8} \text{ or } \underline{1})gg\rangle \\
& + O(v^2)|c\bar{c}({}^1S_0, \underline{8} \text{ or } \underline{1})gg\rangle + \cdots.
\end{aligned} \tag{1}$$

This expansion has also been given in Ref. [13]. But, they missed the last term in their expansion. This term is crucially important for η_c production in γp process which will be shown in the following. For the production of η_c , the contributions to the NRQCD matrix elements from the last three terms of the above expansion are the same order of v^2 according to the NRQCD velocity scaling rules. They are all suppressed by v^4 compared to the contribution from the first term (the color-singlet contribution). However, the color-singlet contributions to η_c photoproduction vanish in the leading order and the next to leading order γg fusion processes (see the following). So, the photoproduction of η_c is purely a color-octet process, even to the next to leading order of QCD calculations. Therefore, the η_c photoproduction processes such as at the HERA, will provide an important test for the color-octet production mechanism. Furthermore, we will show by the following calculations, the dominant contribution to η_c photoproduction comes from the last term of the Fock state expansion in equation (1). For this term, the associated color-octet production matrix element $\langle \mathcal{O}_8^{\eta_c}({}^1S_0) \rangle$ can be related to the matrix element $\langle \mathcal{O}_8^\psi({}^3S_1) \rangle$ by the heavy quark symmetry. And the later color-octet matrix element is important for the color-octet explanation of the prompt J/ψ surplus production at the Tevatron [6,7]. So, the measurement of η_c photoproduction processes is closely associated with the test of the color-octet gluon fragmentation mechanism proposed in [6].

According to the NRQCD factorization formalism [2], the production process $\gamma + g \rightarrow \eta_c + X$ can be expressed as the following form,

$$\sigma(\gamma + g \rightarrow \eta_c + X) = \sum_n F(\gamma + g \rightarrow n + X) \langle \mathcal{O}_n^{\eta_c} \rangle. \tag{2}$$

Here, n denotes the $c\bar{c}$ pair configuration in the expansion terms of Eq. (1) (including angular momentum ${}^{2S+1}L_J$ and color index 1 or 8). $F(\gamma + g \rightarrow n + X)$ is the short distance

coefficient for the subprocess $\gamma + g \rightarrow n + X$. $\langle \mathcal{O}_n^{\eta_c} \rangle$ is the long distance non-perturbative matrix element which represents the probability of the $c\bar{c}$ pair in n configuration evolving into the physical state η_c . The short distance coefficient F can be calculated by using perturbative QCD in expansion of powers of α_s . The long distance matrix elements are still not available from the first principles at present. However, the relative importance of the contributions from different terms in (2) can be estimated by using the NRQCD velocity scaling rules.

From Eq.(1), we can see that the color-singlet contribution to the production of η_c is at the leading order in v^2 . The associated short distance coefficient is given by the subprocess

$$\gamma g \rightarrow c\bar{c}(^1S_0, \underline{1}) + g. \quad (3)$$

This process occurs at the next to leading order in α_s for the γg fusion processes. However, there is no contribution from this process, because it violates C (charge) parity conservation. (The C parities of the two gluon system (constrained in color-singlet) and the $c\bar{c}$ pair in $(^1S_0, \underline{1})$ state are both $+1$, while the C parity of the photon is -1).

The color-octet contributions to the η_c photoproduction come from the leading order and the next to leading order γg fusion processes as shown in Fig.1. At the leading order in α_s , the subprocess is $2 \rightarrow 1$ (Fig.1(a)),

$$\gamma g \rightarrow c\bar{c}(^1S_0, \underline{8}). \quad (4)$$

For this process, we readily have [9]

$$\sigma(\gamma + g \rightarrow c\bar{c}(^1S_0, \underline{8}) \rightarrow \eta_c) = \frac{\pi^3 e_c^2 \alpha \alpha_s}{m_c^3} \delta(\hat{s} - 4m_c^2) \langle \mathcal{O}_8^{\eta_c}(^1S_0) \rangle, \quad (5)$$

where \hat{s} is the invariant mass squared of the partonic process. m_c is the charm quark mass, and we have approximated the charmonium bound state mass of η_c by $2m_c$.

At the next to leading order in α_s , the subprocesses are $2 \rightarrow 2$ (Fig.1(b) and Fig.1(c)). These processes contain the contributions from

$$\gamma + g \rightarrow c\bar{c}({}^1S_0, \underline{8}) + g, \quad (6)$$

$$\gamma + g \rightarrow c\bar{c}({}^3S_1, \underline{8}) + g, \quad (7)$$

$$\gamma + g \rightarrow c\bar{c}({}^1P_1, \underline{8}) + g. \quad (8)$$

The last two subprocesses only have contributions from diagrams shown in Fig.1(b). The diagrams shown in Fig.1(c) do not contribute to $({}^3S_1, \underline{8})$ and $({}^1P_1, \underline{8})$ subprocesses because of C parity conservation, and they only contribute to the $({}^1S_0, \underline{8})$ production subprocess.

The cross sections for these $2 \rightarrow 2$ subprocesses can be expressed as the following form,

$$\frac{d\sigma}{d\hat{t}}(\gamma + g \rightarrow \eta_c + X) = \frac{1}{16\pi\hat{s}^2} F({}^{2S+1}L_J^{(8)}) \langle \mathcal{O}_8^{\eta_c}({}^{2S+1}L_J) \rangle, \quad (9)$$

where $\hat{t} = (z - 1)\hat{s}$, and z is defined as $z = p \cdot k_{\eta_c} / p \cdot k_\gamma$ with p , k_{η_c} , k_γ being the momenta of the proton, the outgoing η_c and the incident photon respectively. For the short distance coefficients F of the subprocesses ${}^3S_1^{(8)}$ and ${}^1S_0^{(8)}$, we readily have [9,14]

$$F({}^3S_1^{(8)}) = \frac{80(4\pi)^3(2m_c)^2 e_c^2 \alpha_s^2}{3(\hat{s} + \hat{t})^2(\hat{s} + \hat{u})^2(\hat{t} + \hat{u})^2} (\hat{s}^2 \hat{t}^2 + \hat{s}^2 \hat{t} \hat{u} + \hat{s}^2 \hat{u}^2 + \hat{s} \hat{t}^2 \hat{u} + \hat{s} \hat{t} \hat{u}^2 + \hat{t}^2 \hat{u}^2), \quad (10)$$

$$F({}^1S_0^{(8)}) = \frac{6(4\pi)^3 \hat{s} \hat{u} \alpha_s^2}{m_c \hat{t} (\hat{s} + \hat{t})^2 (\hat{s} + \hat{u})^2 (\hat{t} + \hat{u})^2} (4\hat{s}^4 + 8\hat{s}^3 \hat{t} + 8\hat{s}^3 \hat{u} + 13\hat{s}^2 \hat{t}^2 + 26\hat{s}^2 \hat{t} \hat{u} + 13\hat{s}^2 \hat{u}^2 + 10\hat{s} \hat{t}^3 + 28\hat{s} \hat{t}^2 \hat{u} + 26\hat{s} \hat{t} \hat{u}^2 + 8\hat{s} \hat{u}^3 + 5\hat{t}^4 + 10\hat{t}^3 \hat{u} + 13\hat{t}^2 \hat{u}^2 + 8\hat{t} \hat{u}^3 + 4\hat{u}^4). \quad (11)$$

Here, \hat{s} , \hat{t} and \hat{u} satisfy the relation $\hat{s} + \hat{t} + \hat{u} = 4m_c^2$. For the ${}^1P_1^{(8)}$ subprocess, we calculate the short distance coefficient and get

$$F({}^1P_1^{(8)}) = \frac{320(4\pi)^3 e_c^2 \alpha_s^2}{3(\hat{s} + \hat{t})^3 (\hat{s} + \hat{u})^3 (\hat{t} + \hat{u})^3} (\hat{s}^6 \hat{t} + \hat{s}^6 \hat{u} + 4\hat{s}^5 \hat{t}^2 + 4\hat{s}^5 \hat{t} \hat{u} + 4\hat{s}^5 \hat{u}^2 + 7\hat{s}^4 \hat{t}^3 + 11\hat{s}^4 \hat{t}^2 \hat{u} + 11\hat{s}^4 \hat{t} \hat{u}^2 + 7\hat{s}^4 \hat{u}^3 + 7\hat{s}^3 \hat{t}^4 + 14\hat{s}^3 \hat{t}^3 \hat{u} + 16\hat{s}^3 \hat{t}^2 \hat{u}^2 + 14\hat{s}^3 \hat{t} \hat{u}^3 + 7\hat{s}^3 \hat{u}^4 + 4\hat{s}^2 \hat{t}^5 + 11\hat{s}^2 \hat{t}^4 \hat{u} + 16\hat{s}^2 \hat{t}^3 \hat{u}^2 + 16\hat{s}^2 \hat{t}^2 \hat{u}^3 + 11\hat{s}^2 \hat{t} \hat{u}^4 + 4\hat{s}^2 \hat{u}^5 + \hat{s} \hat{t}^6 + 4\hat{s} \hat{t}^5 \hat{u} + 11\hat{s} \hat{t}^4 \hat{u}^2 + 14\hat{s} \hat{t}^3 \hat{u}^3 + 11\hat{s} \hat{t}^2 \hat{u}^4 + 4\hat{s} \hat{t} \hat{u}^5 + \hat{s} \hat{u}^6 + \hat{t}^6 \hat{u} + 4\hat{t}^5 \hat{u}^2 + 7\hat{t}^4 \hat{u}^3 + 7\hat{t}^3 \hat{u}^4 + 4\hat{t}^2 \hat{u}^5 + \hat{t} \hat{u}^6). \quad (12)$$

Have provided all the cross section formulas for the subprocesses, we may estimate the η_c production rate in the photoproduction processes. Because there is no color-singlet contribution in our calculation, all of the following numerical results are for the color-octet

production. The results depend on the numerical values of α_s , m_c , and the factorization scale Q . We use $\alpha_s(m_c^2) = 0.3$, $m_c = 1.48\text{GeV}$ and $Q^2 = (2m_c)^2$. For the parton distribution function of the proton, we use the GRV LO parametrization [15].

In Fig.2, we show the η_c photoproduction rate via the $2 \rightarrow 1$ subprocess, in which the intermediate color-octet state is $^1S_0^{(8)}$. Following the heavy quark spin symmetry, we estimate the associated color-octet matrix element $\langle \mathcal{O}_8^{\eta_c}(^1S_0) \rangle$ to be

$$\langle \mathcal{O}_8^{\eta_c}(^1S_0) \rangle \approx \langle \mathcal{O}_8^\psi(^3S_1) \rangle = 1.06 \times 10^{-2} \text{GeV}^3. \quad (13)$$

The value of the color-octet matrix element $\langle \mathcal{O}_8^{J/\psi}(^3S_1) \rangle$ follows the fitted value in [16] by comparing the theoretical prediction of direct J/ψ production to the experimental data at the Tevatron. The $2 \rightarrow 1$ process contributes the η_c photoproduction in the forward direction ($z \sim 1$ and $p_T^2 \approx 0$). In Ref. [9], the authors calculated the J/ψ production in the forward direction and found some conflicts between the NRQCD prediction and the experimental measurements. So, the numerical result shown in Fig.2 may be also questioned by the same problem. However, we note that in the case of J/ψ production, the conflicts with experiment focus on the values of the color-octet matrix elements $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ and $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$, which are determined by the lower p_T distribution data of direct J/ψ production at the Tevatron [7]. The lower p_T J/ψ production may be strongly affected by the initiate and final states gluons radiation, which makes the determination of the matrix elements $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ and $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$ from the Tevatron data very poor [11]. However, we see that the η_c photoproduction in the forward direction of Fig.2 depends on the value of the color-octet matrix element $\langle \mathcal{O}_8^{\eta_c}(^1S_0) \rangle$, which is related to the color-octet matrix element $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$ by the heavy quark symmetry. And the later color-octet matrix element is determined by the large p_T J/ψ production data at the Tevatron, which can be viewed as a more reliable estimate because in the large p_T region gluon fragmentation is dominant.

In Fig.3-4, we show the photoproduction cross section of η_c via the $2 \rightarrow 2$ subprocesses at the HERA, to which there are three color-octet subprocesses contributions ($^1S_0^{(8)}$, $^3S_1^{(8)}$ and $^1P_1^{(8)}$ subprocesses). In Fig.3, we plot the differential cross section $d\sigma/dz$ distribution

for η_c production. In Fig.4, we plot $d\sigma/dp_T^2$ distribution. In these two figures, there are contributions from $^3S_1^{(8)}$ and $^1P_1^{(8)}$, for which we estimate the associated color-octet matrix elements by using the naive NRQCD velocity scaling rules,

$$\langle \mathcal{O}_8^{\eta_c}(^3S_1) \rangle \approx \langle \mathcal{O}_8^{\eta_c}(^1S_0) \rangle, \quad (14)$$

$$\langle \mathcal{O}_8^{\eta_c}(^1P_1) \rangle \approx \langle \mathcal{O}_8^{\eta_c}(^1S_0) \rangle. \quad (15)$$

In these two figures, we impose a cut on z and P_T^2 : $0.2 < z < 0.8$ for Fig.4 and $P_T^2 > 1\text{GeV}^2$ for Fig.3. From these two figures, we can see that the contribution from the $^1S_0^{(8)}$ subprocess dominates over the contributions from the other two processes. Furthermore, the $^1S_0^{(8)}$ contribution to the z distribution of the differential cross section (Fig.3) rises rapidly as z increases. We note that for the J/ψ photoproduction process, the z distribution of the cross section from the color-octet contribution has also this property [9]. However, this is not consistent with the experimental results [8]. This conflict between the color-octet model prediction and the experiment invoke a lot of interesting for the further investigations on the J/ψ photoproduction [11,12]. In [11], the authors noted that the initiate and final state gluon radiation may strongly affect the determinations of the color-octet matrix elements $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ and $\langle \mathcal{O}_8^\psi(^3P_J) \rangle$ from the direct J/ψ production data. This is because the determinations of these matrix elements are sensitive to the data of J/ψ production in the lower p_T region, in which the gluon radiation effects are important. These matrix elements are also the dominant contributions to the J/ψ photoproduction in the high z region. So, by involving these effects, they lowered down the determined values of these color-octet matrix elements, and found they would lead to a consistent description of the z distribution for the J/ψ photoproduction. However, these arguments do not affect the η_c photoproduction of our results in Fig.3. This is because the dominant contribution in Fig.3 comes from the color-octet matrix element $\langle \mathcal{O}_8^{\eta_c}(^1S_0) \rangle$, which can be safely related to the matrix element $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$ by the heavy quark symmetry, and the value of the matrix element $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$ we used is from the fitting to the J/ψ production data in the large p_T [16] region which is weakly affected by the initiate and final states gluon radiation [11] and therefore can be

viewed as a reliable estimate. Another explanation conflict problem emphasizes that the intrinsic k_T may be important for the J/ψ photoproduction [12]. If this is true, our predictions of Fig.3 and Fig.4 for the η_c photoproduction will also be affected by the intrinsic k_T effects. Nevertheless, the measurement of the η_c photoproduction can provide a consistent test of these explanations of the J/ψ photoproduction conflicts [11,12].

For the experimental observation of η_c , one may choose the following decay modes, such as $\eta_c \rightarrow \phi\phi$, $\eta_c \rightarrow \gamma\gamma$ and $\eta_c \rightarrow \rho\rho$, which either have substantial branching ratios or may be easy to detect. From Figs.2-4, we can see that the production rate of η_c in the photoproduction processes is comparable with that of J/ψ , and we hope that the experimental measurement of η_c photoproduction will soon appear.

In conclusion, we calculated the η_c photoproduction under the NRQCD factorization formalism. We find that the color-singlet contribution to this process vanishes at the leading order and the next to leading order QCD calculations. Therefore, this process is a pure color-octet process, and the measurement of this process can provide useful information to clarify the existing conflict between the color-octet prediction and the experimental results on the J/ψ photoproduction.

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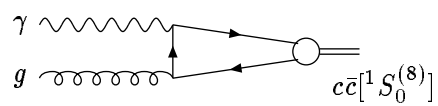
Figure Captions

FIG. 1. The generic Feynman diagrams for the photoproduction of η_c at the leading order (a) and the next to leading order (b) and (c) γg fusion processes. All of these diagrams are for the color-octet processes.

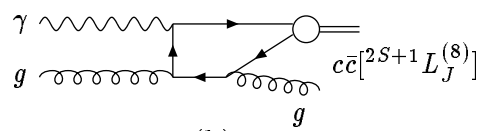
FIG. 2. The cross section of $\gamma + g \rightarrow \eta_c + X$ in the forward direction ($z \sim 1$, $p_T^2 \approx 0$) as a function of the photon-proton c.m. energy $E_{c.m.} = \sqrt{s_{\gamma p}}$.

FIG. 3. The differential cross sections $d\sigma/dz$ for the process $\gamma + p \rightarrow \eta_c + X$ at the HERA as a function of $z \equiv E_{\eta_c}/E_\gamma$, where $\sqrt{s_{\gamma p}} = 100 \text{ GeV}$. The curves denote contributions from the color-octet 3S_1 , 1P_1 , and 3S_1 respectively.

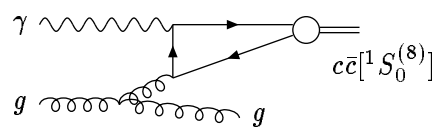
FIG. 4. The differential cross sections $d\sigma/dP_T^2$ for $\gamma + p \rightarrow \eta_c + X$ at the HERA as a function of p_T^2 , where $\sqrt{s_{\gamma p}} = 100 \text{ GeV}$. The curves denote contributions from the color-octet 3S_1 , 1P_1 , and 3S_1 respectively.



(a)



(b)



(c)

